

# Inverse Hyperbolic Functions

## Definitions

$$\operatorname{arcsinh}(x) = \ln(x + \sqrt{x^2 + 1})$$

$$\operatorname{arccosh}(x) = \ln(x + \sqrt{x^2 - 1})$$

$$\operatorname{artanh}(x) = \frac{1}{2} \ln\left(\frac{x+1}{x-1}\right)$$

$$\operatorname{arcoth}(x) = \operatorname{artanh}\left(\frac{1}{x}\right) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$

$$\operatorname{arcsech}(x) = \operatorname{arccosh}\left(\frac{1}{x}\right) = \ln\left(\frac{1}{x} + \frac{1}{x} \sqrt{1-x^2}\right)$$

$$\operatorname{arccsch}(x) = \operatorname{arcsinh}\left(\frac{1}{x}\right) = \ln\left(\frac{1}{x} + \frac{1}{x} \sqrt{x^2 + 1}\right)$$

## Opposite Argument Formulas

$$\operatorname{arcsinh}(-x) = -\operatorname{arcsinh}(x)$$

$$\operatorname{arccosh}(-x) = \operatorname{arccosh}(x)$$

$$\operatorname{artanh}(-x) = -\operatorname{artanh}(x)$$

$$\operatorname{arcoth}(-x) = -\operatorname{arcoth}(x)$$

$$\operatorname{arcsech}(-x) = \operatorname{arcsech}(x)$$

$$\operatorname{arccsch}(-x) = -\operatorname{arccsch}(x)$$

## Identities

$$\sinh(\operatorname{arccosh}(x)) = \sqrt{x^2 - 1}$$

$$\cosh(\operatorname{arcsinh}(x)) = \sqrt{x^2 + 1}$$

$$\sinh(\operatorname{artanh}(x)) = \frac{x}{\sqrt{1-x^2}}$$

$$\cosh(\operatorname{artanh}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\tanh(\operatorname{arcsinh}(x)) = \frac{x}{\sqrt{x^2 + 1}}$$

$$\tanh(\operatorname{arccosh}(x)) = \frac{\sqrt{x^2 - 1}}{x}$$

## Sum Formulas

$$\operatorname{arcsinh}(a) \pm \operatorname{arcsinh}(b) = \operatorname{arcsinh}(a\sqrt{b^2 + 1} \pm b\sqrt{a^2 + 1})$$

$$\operatorname{arccosh}(a) \pm \operatorname{arccosh}(b) = \operatorname{arccosh}(ab \pm \sqrt{a^2 - 1}\sqrt{b^2 - 1})$$

$$\operatorname{artanh}(a) \pm \operatorname{artanh}(b) = \operatorname{artanh}\left(\frac{a \pm b}{1 \pm ab}\right)$$

$$\begin{aligned} \operatorname{arcsinh}(a) \pm \operatorname{arccosh}(b) &= \operatorname{arccosh}(b\sqrt{a^2 + 1} \pm a\sqrt{b^2 - 1}) \\ &= \operatorname{arcsinh}(ab \pm \sqrt{a^2 + 1}\sqrt{b^2 - 1}) \end{aligned}$$

## Equations, $k \in \mathbb{Z}$

$$\sinh(x) = a \rightarrow x = \operatorname{arcsinh}(a) + 2ik\pi$$

$$\cosh(x) = a \rightarrow x = \pm \operatorname{arccosh}(a) + 2ik\pi$$

$$\tanh(x) = a \rightarrow x = \operatorname{artanh}(a) + ik\pi$$

$$\coth(x) = a \rightarrow x = \operatorname{arcoth}(a) + ik\pi$$

$$\operatorname{sech}(x) = a \rightarrow x = \pm \operatorname{arcsech}(a) + 2ik\pi$$

$$\operatorname{csch}(x) = a \rightarrow x = \pm \operatorname{arccsch}(a) + 2ik\pi$$